EECS C145B / BioE C165 Spring 2004: Problem Set V
Due May 4 2004

Please read the sections describing the rules for working in groups and the grading policy in the course introduction handout.

Show all code and plots.

Problem 1 (50 points)

A certain tomographic imaging protocol involves taking 16 projections over 180 degrees. Each projection contains 29 bins. You are given a set of projections in the binary Matlab workspace file:

`http://muti.lbl.gov/145b/sinogram_annulus.mat`

1. Using *radon* calculate the projection geometrical weighting factor matrix $\mathbf{F}$ if you are to reconstruct the image on a 16×16 grid. You should consider using *save* to store this matrix, as it may take a while to generate it. Plot $\mathbf{F}^T\mathbf{F}$ as an image. Describe its structure.

2. Reconstruct the image using the pseudoinverse. Plot the singular value spectrum and choose an appropriate truncation point. Reconstruct using 256 singular vectors and using your chosen truncation point. Compare the two results qualitatively. Explain why truncation helps.

Problem 2 (25+25+15 points)

1. Many experts believe that all mammals get cancer. You are examining 100 "bumblebee bats" (*craseonycteris thonglongyai*), one of the smallest known mammals, for the presence of tumors. What modality would probably be the best to use and why? (In your answer consider, for example: resolution, sensitivity, contrast mechanism, practicality and cost)

2. What would be a good method for determining iron distribution in the animal's brain? Why? Explain the contrast mechanism.

3. A colleague in a competing group is using small animal PET to study a slightly larger bat and is using a tracer labeled with ^{82}Rb. Should you follow your colleague's lead? Why?

Problem 3 (25+25 points)

1. Derive an expression for the sensitivity of an ECT tomograph and compare the sensitivity of SPECT and PET for human brain imaging (see reader).

2. Compare the sensitivity and minimal resolvable feature size (at tomograph center and a few cm away from center) of the MicroPET imager described at:

   `http://www.crump.ucla.edu/user-files/`
   `resprojects/microPET/fr_design.html`

   with the imager proposed in the paper:

   `http://muti.lbl.gov/145b/LBNL-42562.pdf`

Explain the basis for your estimates and comparisons. Hint: Concentrate on crystal width, crystal depth, crystal composition and number of crystals.

Problem 4 (50 points)

Complete the following problems in MRI Basics (in the reader):

1. 4-1 through 4-5. (reader p. 307)

2. 5-1 through 5-5. (reader p. 315)

Optional problems (not graded)

Optional problem 1

You have a program that calculates the pseudoinverse. Show analytically how the same program can be used to solve the weighted least squares problem:

$$\boldsymbol{\mu}_{\text{WLS}} = (\mathbf{F}^T \boldsymbol{\Sigma}^{-1} \mathbf{F})^{-1} \mathbf{F}^T \boldsymbol{\Sigma}^{-1} \mathbf{p}$$

Optional problem 2

You are designing a small animal SPECT system. What type of collimator would you use to:

1. Optimize resolution and sensitivity when the detector area is similar in size to the animal?

   _____

2. Optimize resolution and cost when the detector area is intended for human imaging?

   _____

3. Optimize resolution and field of view when the detector area is intended for human imaging?

   _____

4. Optimize resolution and sensitivity when the detector area is intended for human imaging?

   _____

5. Optimize field of view when only a small region of the detector area (smaller than the animal) is functional?

   _____

Optional problem 3

A rectangular block of lead is 2 cm long and 3 cm wide. Its center is positioned at the center of rotation of an x-ray CT camera.

1. Approximately draw the parallel beam projections for $\boldsymbol{\theta}$ oriented at $0, 45$ and 90 degrees.

2. If the attenuation coefficient of lead is μ per meter, find an expression for the amplitude of the central bin of the 45 degree projection.

Optional problem 4

An image is acquired using a spin-echo imaging sequence, but the values of TR and TE used have been forgotten. Given the information in Figure 1, determine whether TR and TE were long or short, relative to T1 and T2, respectively, when the image in Figure 2 was acquired.

   Check one box:

		Answer
TR short	TE long	
TR long	TE short	
TR long	TE long	
TR short	TE short	

Suppose the fourth blob was the brightest blob. What would you say about TR and TE?

Check one box:

		Answer
TR short	TE long	
TR long	TE short	
TR long	TE long	
TR short	TE short	

Suppose the first and second blobs were bright and the third and fourth dark. What would you say about TR and TE?

Check one box:

		Answer
TR short	TE long	
TR long	TE short	
TR long	TE long	
TR short	TE short	

The equation governing contrast is:

$$\rho(x, y, z) = \rho_0(x, y, z) e^{-\frac{T_E}{T_2}} \left[1 - e^{-\frac{T_R}{T_1}} \right], \tag{1}$$
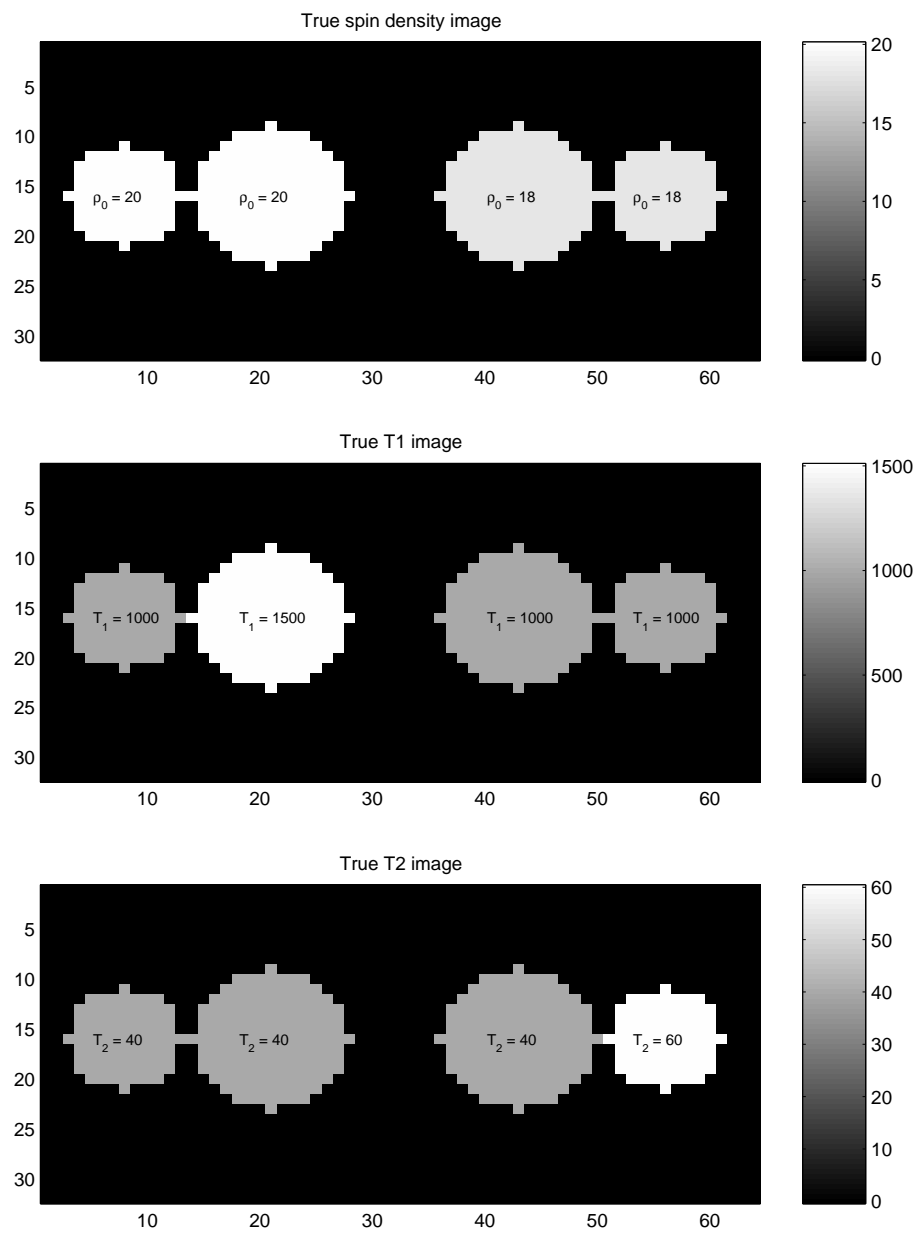
where $\rho_0(x, y, z)$ is the spin-density.
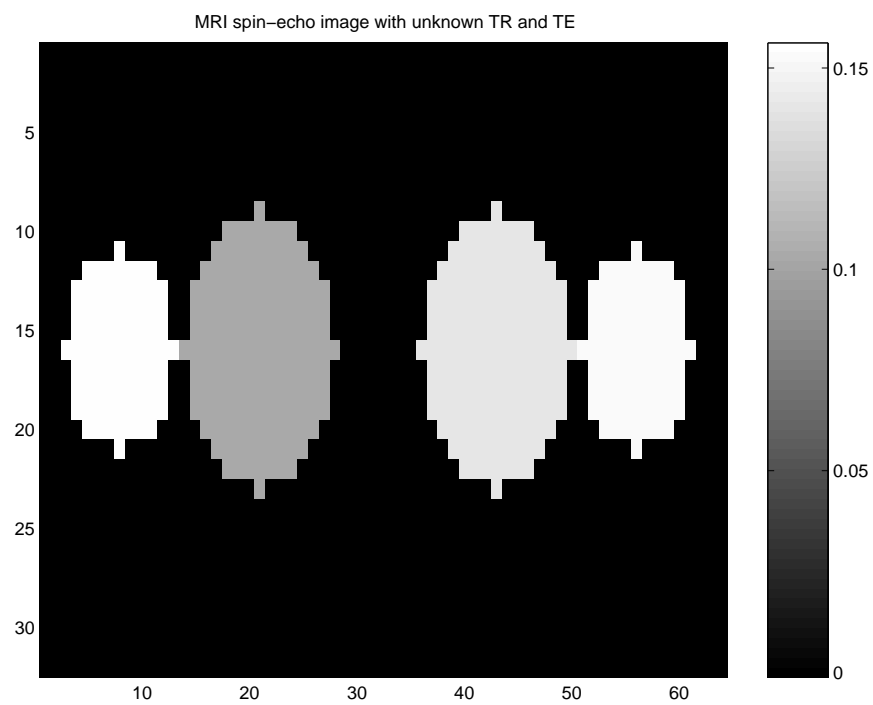
Figure 1: True T1, T2 and spin-density (ρ_0) distributions

Figure 2: Image obtained with TE and TR unknown